Strain Measurement Instrumentation and Technique

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Strain measurement instrumentation and technique

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Methods of measuring strain in the Earth's surface are reviewed briefly. An important aspect of strain measurement which is often overlooked is the effect on the measurement of the environment in which the strainmeter is sited. The site is shown in certain cases to have an important influence on measured strain. The effects of elastic inhomogeneity near a strainmeter are examined including the effects of underground cavity or fissure geometry. Records showing the effects of site inhomogeneity are presented.

Introduction

The first part of this paper is a brief review of the development of strain instrumentation for geophysical purposes. Instrumental details are not discussed for their own sake but only where they relate to some particular advance in conception. The second part of the paper is concerned with instrument siting. For strainmeters this cannot really be separated from instrument design, although many investigators have not concerned themselves with this problem. The rock to which a strainmeter is attached is virtually an integral part of the instrument. The site rock can alter the magnitude of the strains it is desired to measure and introduce spurious signals which have their origin in other strain components.

1. Steel and quartz rod systems

Geophysical strain measurement appears to have started in Japan in 1888 (Milne 1888). Later work was carried out by Oddone (1900). Both used rather crude systems sited on the surface of the ground.

As a properly conceived geophysical technique, however, strain measurement was started by Benioff in 1935 working in California. He used a steel length standard but placed it underground for thermal stability. To increase sensitivity he made the instrument 25 m long instead of the 2 to 3 m used previously. With this early instrument he had a directly read output employing a travelling microscope. But he also achieved a high sensitivity continuously recording output by using a magnetic velocity transducer of a type that had previously been developed for displacement seismographs. (He was therefore recording a time differential of strain.) This rendered his instrument capable of detecting medium-period waves from distant earthquakes, but did not allow him to record strains of long-period seismic or tidal periods. More important than his practical achievement was his correct introduction into the literature of the concept of linear strain as it relates to the propagation of seismic waves.

In his initial paper, however, he failed to fully understand the relation between horizontal linear strain and other components. All 'geophysical' information available to strain measurement may be measured using linear strainmeters in a horizontal plane (King 1971). Benioff's discussion of volumetric and vertical strainmeters is not therefore very useful and some of his proposed designs for vertical strainmeters would in fact measure a spatial derivative of strain and not true strain. Mistakes of this sort made by Benioff in 1935 are not of any importance. It is, however, unfortunate that various sorts of alleged strainmeters other than linear strainmeters continue to be constructed.

Apart from getting strain theory partly correct, a second important advance of Benioff was to appreciate, if not fully, the importance of good strainmeter siting. He considered that a strainmeter should be sited 'deep' in 'hard' rock. His appreciation that the rock is a part of the instrument was correct, but his assumption that *deep* burial in *hard* rock would prove the best means of both ensuring a good coupling to strains of 'global' origin and reducing the effects of surface generated noise are not correct.

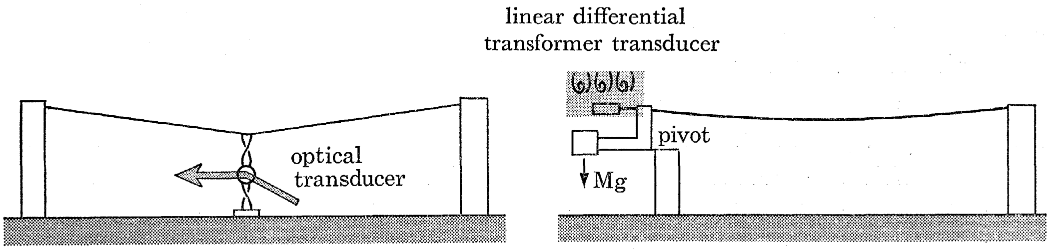
In 1954 and 1959 Benioff published details of a much improved instrument. The steel rod of his 1935 device was replaced by a quartz rod to minimize the effect of temperature coefficient of the rod, and transducers which gave outputs in terms of strain were introduced. One transducer used a system of thin filaments to rotate the mirror of a photographically recording optical level system and a frequency discriminator (or resonant) bridge type of capacitive transducer provided the other. Systems of this type are inherently nonlinear over any wide range of amplitude and liable to variations of gain with time. This rendered *continuous* calibration essential. Benioff achieved this with a magnetostrictive element. *Initial* calibration was carried out interferometrically.

Benioff solved many problems of strain measurement and highlighted many unsolved problems. Other investigators who have followed his direct lead are too numerous to review here in detail (see, however, Bilham 1970). However, attempts to improve calibration using a laser interferometer (V. B. Gerard, private communication) and the optical calibration of the allegedly portable instrument of Blayney & Gilman (1965) are notable.

Major and others have succeeded in installing quartz rod systems around nuclear test sites in Nevada and Amchitka in surface trenches to observe long period static strains associated with nuclear testing (Major *et al.* 1969; Romig *et al.* 1969).

2. Invar wire and rod systems

In Japan in the early 1950s Sassa developed an instrument of quite different design to the Benioff instrument (See Ozawa 1961*a*). Thin invar wire was used instead of a rigid rod. Invar has a slightly superior temperature coefficient to quartz, but the experience of surveyors (e.g. Hotine 1939) probably suggested to American investigators that it was not a suitable material for sensitive strain measurement owing to erratic creep behaviour. Sassa's design is shown in figure 1*a*. It was capable of high sensitivity and was extremely simple. This simplicity, however, was marred by the instability of the gain and the difficulty of calibration. Sassa also used a rod system and a roller carrying the mirror of an optical lever. Ozawa (1961*a*) compared the Sassa instrument with an instrument of his own which uses a horizontal pendulum as a displacement



(*a*) Figure 1. Wire strainmeters. (*b*) transducer. The horizontal pendulum system was shown to be superior to the other methods and capable of good calibration, but left something to be desired as regards convenience and robustness.

A spectacular advance in wire strainmeter design was provided by the constant tension wire strainmeter described by Sydenham in 1969 (figure 1*b*). An important feature of his design was the use of a differential transformer or mutual inductance bridge (in a non-resonant condition) as a transducer. An inductive bridge may be made linear over a wide range and is very insensitive to contaminants. (This is because dirt in general has a low permeability compared with materials of ferro-magnetic permeability used in its construction. This does not apply to dielectric constants of contaminants in a capacitive system.)

The constant tension system has been adopted by a number of groups and has shown itself capable of being adapted to produce portable systems of good calibration (Sydenham, this volume; Gerard, this volume; Bilham & King 1970).

3. Optical systems

The uncertainty of the dimensional stability of solid length standards and the limitation of instrument length imposed by the use of quartz rods attached end to end or long spans of invar wire led to an interest in the use of optical systems. In particular the advent of the gas laser in 1954 offered new possibilities (Gordon, Zeiger & Townes 1954; Cook, Marussi & Rowley 1965).

The first laser system was constructed by Van Veen, Savino & Alsop in 1966. He used two mutually perpendicular lasers with their end mirrors attached to the rock. Changes of cavity length caused frequency changes in the lasers. The frequency difference between the two was found by beating the optical outputs in a photomultiplier. The frequency of the electrical output of the photomultiplier provided a measure of shear strain in axes orientated at 45° to the laser axes.

For practical reasons it was not possible, and is still impossible, to construct single mode lasers much over 1 m in length, with the result that later laser systems have not followed the Van Veen system of coupling the laser cavity directly on to the rock. All subsequent systems have used a long passive interferometer powered by an external laser.

Three methods of measuring strain are possible with this configuration: fringe following, fringe counting and beat frequency. Vali & Bostrom (1968) developed the first long path laser strainmeter and described a system 1020 m in length in 1968. King & Gerard (1969) developed a fringe counting system, and Barger & Hall (1969) a beat frequency system. Both of the latter used the stability of frequency of an atomic transition as a standard. The use of an absorption system using a methane cell and the 3.39 μm line adopted by Barger & Hall (better than 1 in 10^{11}) was, however, considerably better than the Lamb dip stabilization system used by King (better than 1 in 10^7).

The most interesting laser system from the geophysical perspective is that of Berger (1969). He has now developed fringe counting interferometers of the order of 1 km in length sited above ground level. They produce excellent results and indicate that even in rather indifferently chosen sites an instrument of that length is long compared to the wavelength of surface generated strain noise. Pier stability remains a problem, but if this is satifactorily overcome then the device will prove a useful, if expensive, method of measuring secular strain.

4. Geophysical information derived from strain measurement

Considerable and varied effort has been expended on the instrumental aspects of strainmeter design. Principle virtues in achieving a good design have been seen as high sensitivity, accurate instrument calibration and good long-term stability. Yet despite this effort, the geophysical information that has been forthcoming from strainmeters has been limited. Principle among their successes was the part they played in revealing free oscillations of the Earth (Benioff 1961), but the data which has subsequently been analysed in detail (by, for example, F. A. Dahlen) has been derived from other types of instrument. The existence of strain steps was also demonstrated with the aid of strainmeters, but despite extensive work around nuclear test zones (Romig *et al.* 1969) and some work by Berger in California, there has been little advance since 1965 (Press 1965). Earth tides of sectoral and tesseral species (semidiurnal and diurnal species) and their sea and ocean loading effects have been observed on many strainmeters, but little useful interpretation has yet been carried out. Work has been done by Major *et al.* (1964), Berger (this volume), Levine (this volume), Ozawa (1955, 1961*b*, 1965) and Bilham *et al.* (1972).

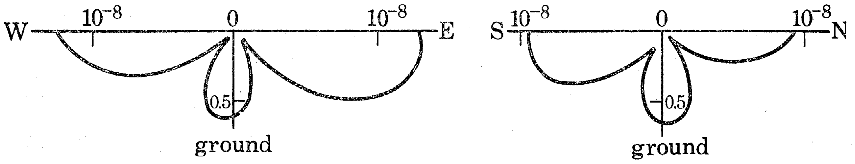


Figure 2. Observed variation of linear strain as a function of vertical angle for the M_2 tidal component. (From Ozawa 1957.)

A singular failure of strainmeter design groups has been a lack of concern about instrument siting. There has been a tacit assumption that a linear strainmeter in a particular orientation measures a strain representative of that orientation. This is by no means necessarily the case for reasons which are examined in the following discussion. Only Ozawa has published sufficient data for comments to be made about his site. Using various instruments he compared the theoretical with the observed tides in various azimuths and as a function of the angle to the vertical (Ozawa 1961*b*). His results are reproduced in figure 2. It should be noted that the patterns both in the west–east and south–north direction are asymmetrical about the vertical axis. The degree of asymmetry is considerable; greater than 20 % of the strain amplitude. At tidal periods, his instruments were effectively at the Earth's surface (a stress-free surface). At a stress-free surface in a uniform or horizontally stratified medium strain must be symmetrical about the vertical axis (King 1971). The cause of the asymmetry of Ozawa's results therefore can only be attributed to two possible effects. Either his instrumental calibration was bad or there were considerable inhomogeneities in the region of his site.

5. Measurement of strain in an inhomogeneous strain field

To understand the relation between a strainmeter and its site it is useful first to examine the relation between the definition of strain and the way in which it is measured.

A linear extension along, say, the x direction may be defined as the fractional change of length Δu of some given length Δx:

$$\text{linear extension} = \Delta u/\Delta x. \tag{1}$$

Linear strain is the differential version of equation (1), i.e.

$$e_{xx} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{\partial u}{\partial x}. \tag{2}$$

Clearly a strainmeter cannot be of infinitesimal length nor is it desirable that it should be. It has been clear to many who have studied elasticity (e.g. Love 1892) that homogeneous strain loses its meaning at a molecular scale. By similar reasoning, measured strain in a pebble beach has meaning over spans large compared to pebble sizes, or small compared to pebble sizes, but not over intermediate spans. This is an obvious example of inhomogeneity. In a strainmeter site region inhomogeneities may also be present but their dimensions and causes are less obvious.

The necessary condition of an extensometer to measure a strain value may be defined by saying that changes in the position of the strainmeter, but not its orientation, by distances of δx, δy and δz of the same magnitude as the instrument length Δx should negligibly alter the magnitude of the strain measured. That is to say:

$$\text{measured extension,} \quad \epsilon_{xx} = \frac{\Delta u}{\Delta x} = \frac{1}{\Delta x}\int_0^{\Delta x} \frac{\partial u}{\partial x}\, \mathrm{d}x \tag{3}$$

$$|\epsilon_{xx}| \gg \left| \frac{1}{\Delta x}\int_0^{\delta x}\int_0^{\Delta x} \frac{\partial^2 u}{\partial x^2}\, \mathrm{d}x\, \mathrm{d}x \right| \tag{4}$$

$$|\epsilon_{xx}| \gg \left| \frac{1}{\Delta x}\int_0^{\delta y}\int_0^{\Delta x} \frac{\partial^2 u}{\partial x \partial y}\, \mathrm{d}x\, \mathrm{d}y \right| \tag{5}$$

$$|\epsilon_{xx}| \gg \left| \frac{1}{\Delta x}\int_0^{\delta z}\int_0^{\Delta x} \frac{\partial^2 u}{\partial x \partial z}\, \mathrm{d}x\, \mathrm{d}z \right|. \tag{6}$$

There are two possible causes of strain varying in ways that violate these relations. One is the presence of propagating seismic waves of wavelength comparable to the instrument length, and the second is due to spatial variations of elastic properties with wavelengths of the order of the instrument length. The first is generally of little importance for geophysical strain measurement since propagating waves of lengths comparable to strainmeter lengths have seismic frequencies of small interest. The effect of spatial changes of elastic properties is more serious since it affects the magnitudes of observed strains at any frequency. Although modification of measured strain magnitude may be considered the primary effect, it is also likely to be accompanied by strains originating in one direction being measured in another direction. This cross-coupling provides a means of recognizing the presence of inhomogeneities from direct examination of the records without recourse to Fourier transform.

6. Strain amplitude changes due to elastic inhomogeneities

Figure 3 shows strainmeters sited on inhomogeneous ground. If certain assumptions are made about the inhomogeneities, it is possible to assess the likely error of strain magnitude measurement that can occur.

Consider first that the elastic moduli are constant in z–y planes, and that the material is unstrained in the same plane (uniaxial stress assumption). The reciprocal of the Young modulus is allowed to vary sinusoidally with x such that

$$\frac{1}{E} = \frac{1}{E_0} + \frac{\sqrt{2}}{E_{\mathrm{r}}} \cos kx.$$

The Poisson ratio ν is considered constant. (It is convenient to use the reciprocal of the Young modulus E. Any profile of this quality can be Fourier synthesized from the expressions given. The longest wavelength component, however, is always likely to be most significant, therefore higher components are ignored and Fourier integrals omitted.)

The mean reciprocal of Young modulus is $1/E_0$, the r.m.s. variation in its value is $1/E_r$, the wavenumber is k ($k = 2\pi/\lambda$ where λ is the wavelength), the mean strain over a distance long compared to λ is $\overline{e_{xx}}$, and x_0 is the centre of the instrument. The measured extension will be

$$\epsilon_{xx} = \frac{\Delta u}{\Delta x} = \overline{e_{xx}}\left[1 - \frac{2\sqrt{2}}{k\Delta x}\left(\frac{E_0}{E_r}\right)\sin\frac{k\Delta x}{2}\cos kx_0.\right]. \tag{7}$$

The r.m.s. fraction error of observed strain to the 'mean strain' $\overline{e_{xx}}$ is

$$\text{r.m.s. error} = \frac{2}{k\Delta x}\left(\frac{E_0}{E_r}\right). \tag{8}$$

This one-dimensional model of inhomogeneities might appear to be so idealized as to be of limited application. If, however, we assume that the material is not discontinuous (no large fissures or cavities) and are prepared for our estimate of error to itself be in error by up to 30 % then the model covers most likely inhomogeneity geometries (and some unlikely ones). It is not a worst case calculation. The model assumes that the material is free to expand in the z–y direction. By a combination of constraint in the regions of high Young modulus, lack of constraint in the regions of the low Young modulus and variations of Poisson ratio it is possible to conceive of the error being greater by 30 %. If the inhomogeneities are small in lateral extent compared to their length then the error could be considerably less. For most inhomogeneities this is unlikely to be sufficient to reduce the error by more than 30 %.

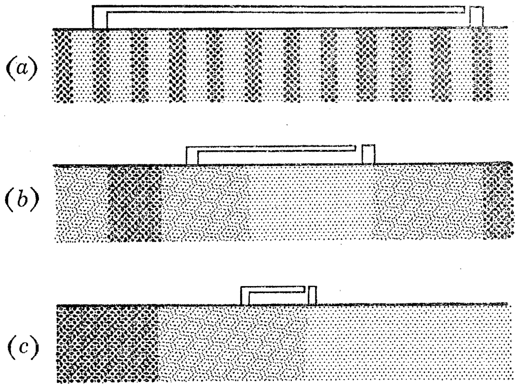


FIGURE 3. Strainmeters mounted on inhomogeneous ground.

Three examples illustrate quantitatively the effect of likely inhomogeneities. If a 10 m strainmeter is mounted on rock in which the Young modulus varies by 20 % with a wavelength of 1 m (figure 3*a*) the strain measurement error would be less than 1 %. This is similar to the situation of mounting a strainmeter on 0.5 m stone end mounts. If, on the other hand, this variation had a wavelength of 20 m an error of 10 % could occur (figure 3*b*). If the instrument is sited in a region where inhomogeneities have wavelengths long compared to the instrument (figure 3*c*), equation (7) becomes

$$\epsilon_{xx} = \overline{e_{xx}}\left[1 - \frac{\sqrt{2}\,E_0}{E_r}\cos kx_0\right]. \tag{9}$$

The difference between the observed strain and $\overline{e_{xx}}$, the 'mean strain', under these conditions varies with the reciprocal of the Young modulus. Variations of this quantity of 20 % will cause measured strain errors of 20 %.

A particularly significant type of inhomogeneity is the tunnel in which a strain measurement is being made. Provided that the measurement is coaxial with the tunnel the tunnel has no effect. From calculations of Panek (1966) it can be shown that a strain measurement in the *x*-direction diametrically across a horizontal tunnel orientated along the *y*-axis will give

$$\epsilon_{xx} = 3\,\overline{\epsilon_{xx}}. \tag{10}$$

Thus strain measurement across a tunnel is amplified three times.

7. Cross-coupling between strain components

The discussion so far has been concerned only with the way in which inhomogeneities in the direction in which a strainmeter is orientated affect the strain magnitudes measured in that direction. Inhomogeneities cause another very important effect. This is cross-coupling between components. Figure 4 shows an example of a simple mechanism whereby this can occur. The vertical fissure is assumed to be deep compared with the dimensions of the strainmeter which is very close to the surface and to the edge of the slot. A strainmeter orientated in the *x*-direction will measure strain in the *y*-direction with a magnitude:

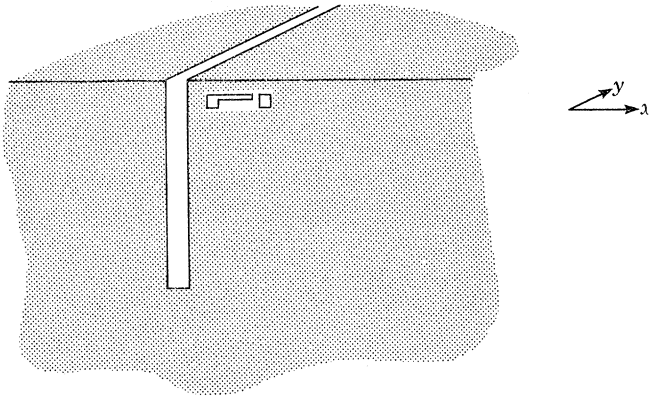


FIGURE 4. Cross-coupling between linear strains in the *x*- and *y*-directions may be caused by the presence of a slot. Other less extreme inhomogeneities of rock properties will also cause cross-coupling.

$$e_{xx} = -\nu\,\overline{e_{yy}}. \tag{11}$$

Under these conditions there will be no observed strains due to strains originating in the *x*-direction. The instrument is cross-coupled to strains in the *y*-direction and decoupled from strains in the *x*-direction. If the slot is filled with material cross-coupling will still occur but with a diminished coefficient. Strains originating in the *x*-direction will then be observed but with altered magnitude. The conditions of an open slot is a worst case. Cross-coupling coefficients greater than the Poisson ratio are therefore not to be expected.

An interesting example of cross-coupling is provided by the effect of the Earth's surface on strain observations. This is discussed by King (1971), where he shows that the result of the free surface is to leave only three independent strain variables observable by geophysical strainmeters.

These are available in a horizontal plane. A vertical strain is also present but is dependent only on the horizontal areal strain.

$$e_{zz} = -\frac{\nu}{1-\nu}(e_{xx}+e_{yy}). \tag{12}$$

The vertical strain results from cross-coupling. This is the strain which would be measured by a vertical borehole strainmeter. If the strainmeter were sited vertically in a horizontal tunnel however, there would be an additional cross-coupling effect due to the tunnel and an amplification effect similar to that described in connexion with equation (10). The measured strain would be

$$e_{zz} = -\left(\overline{e_{xx}}+\frac{\nu}{1-\nu}\overline{e_{yy}}\right). \tag{13}$$

8. The observed effects of inhomogeneities

The signal which is most easily studied for the effects of variations of rock properties is the tidal signal. With a number of collinear instruments sited in different parts of the same tunnel an assessment of the effects of inhomogeneities can be made.

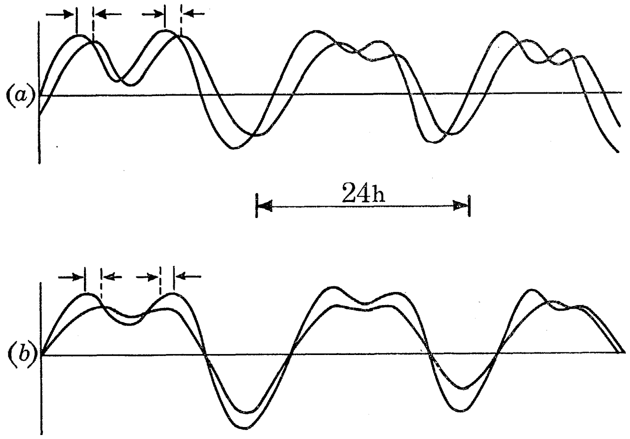


Figure 5. Modification of tidal records by inhomogeneities in the site region.

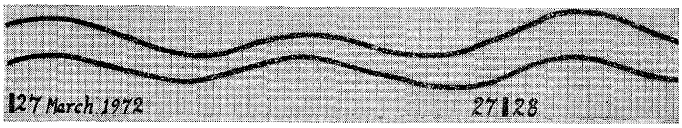


Figure 6. A sample of strainmeter records showing the effects of site inhomogeneity. The two intruments were approximately 500 m apart. Similar effects have been observed on instruments at separations of 2 m.

The direct effect of inhomogeneities on strain amplitudes is not directly obvious on the records owing to calibration differences between instruments. Another effect, however, can clearly be seen. This is 'peak shifting' and is shown in the cartoon record figure 5*b*. It is different from phase shifting which would produce effects of the type shown in figure 5*a*.

Peak shifting is caused by modification of the relative amplitudes of diurnal and semidiurnal strain as a result of cross-coupling of strain from other components.

A sample of peak shifted record from two instruments sited 500 m apart in a disused railway tunnel in Yorkshire, U.K., is shown in figure 6. It is typical of the differences between the two instruments at full or new Moon.

It might be expected that cross-coupling could also give rise to phase shifts but this has not been observed on the raw data although it may appear when the data is Fourier analysed.

The observed peak shifting effect, however, is large, and must result from changes of magnitude of tidal components of at least 10%. From the preceding discussion of the effect of rock inhomogeneities it is clear that this can only result from significant variations of rock properties in the site region.

9. CONCLUSIONS

It is now possible to construct inexpensive short baseline (10 m) strainmeters. Their calibration can be within 2 or 3% and they are easily portable. It is, however, clear that instruments of this length must be sited with care. If they are sited in tunnels, measurements across the tunnel either vertically or horizontally should be avoided since large errors can result. Measurements axially along tunnels are preferable but may be subject to effects due to inhomogeneities. These produce magnitude and possibly phase errors which can be of the order of several per cent and therefore exceed errors in instrument calibration. If short instruments are to provide data for studies in which very accurate amplitude and phase data is required, ways of selecting sites free from the 'spatial noise' resulting from inhomogeneities must be found.

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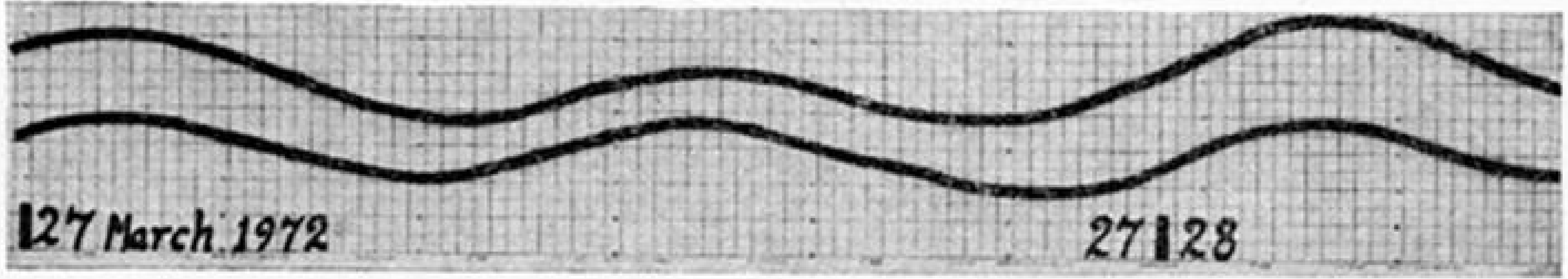

IGURE 6. A sample of strainmeter records showing the effects of site inhomogeneity. The two intruments were approximately 500 m apart. Similar effects have been observed on instruments at separations of 2 m.